C 016

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2009.

THIRD SEMESTER

MA 1201 — MATHEMATICS – III

(Common to All Branches (Except Bio Medical))

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Form a partial differential equation by eliminating the arbitrary constants a and b from $z = (x+a)^2 + (y-b)^2$. $x+a = p/2$ (1) $y-b = q/2$ (1) $\Rightarrow p^2+q^2 = 4z$ (1)

2. Find the complementary function of $(D^2 + 2DD' + D'^2)z = xy$. $m = -1, -1$ (1) $\phi_1(y-x) + x\,\phi_2(y-x)$ (1)

3. State the conditions for $f(x)$ to have Fourier series expansion. (2)

4. Find b_{10} in the Fourier expansion of $x^2 - 2$ in the range $|x| \le 3$. $= 0$ (2)

5. Write down the three possible solutions of one dimensional heat equation. $(Ax+B)$, $(A\cos(\lambda x) + B\sin(\lambda x))\,e^{-\alpha^2\lambda^2 t}$, $(Ae^{\lambda x} + Be^{-\lambda x})\,e^{\alpha^2\lambda^2 t}$ (2)

6. Classify : $x^2 u_{xx} + 2xy \cdot u_{xy} + (1+y^2)u_{yy} - 2u_x = 0$. $= -4x^2$ (1) elliptic (1)

7. State the Fourier Integral theorem. $\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t)\, e^{is(x-t)}\, dt\, ds$ (2)

8. Prove that $\frac{d}{ds}\{\bar{f}_C(S)\} = -F_S\{xf(x)\}$ where $F_S\{f(x)\} = \bar{f}_S(S)$. $= \int_0^{\infty} f(x)(-x\sin(sx))\,dx$ (1) $= -\int_0^{\infty} x f(x)\sin(sx)\,dx$ (1)

9. Find the Z – transform of unit sample sequence. $\delta(n) = 1,\ n = 0;\ 0,\ n \ne 0$ (1) $Z(\delta(n)) = 1$ (1)

10. State the initial value theorem of Z – transform.
$f(0) = \lim_{z\to\infty} Z[f(n)]$ (2)

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Find the differential equation of all spheres of the same radius c having their centres on the yoz–plane. (8)

(ii) Find the general solutions of $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$. (8)

Or

(b) (i) Find the complete integral of $\sqrt{p} + \sqrt{q} = 2x$. (8)

(ii) Solve $(D^2 - 3DD' + 2D'^2)z = e^{3x+4y} + \sin(4x - 3y)$. (8)

12. (a) (i) Find the Fourier series of $\sin x$ in $-\pi < x < \pi$. (8)

(ii) Obtain the first three harmonics for the data (8)

x:	0	1	2	3	4	5
y:	4	8	15	7	6	2

Or

(b) (i) Prove that in $0 < x < l$,

$$x = \frac{l}{2} - \frac{4l}{\pi^2}\left[\cos\frac{\pi x}{l} + \frac{1}{3^2}\cos\frac{3\pi x}{l} + \frac{1}{5^2}\cos\frac{5\pi x}{l} + \ldots\right].$$ (8)

(ii) Find the complex form of the Fourier series of $f(x) = e^{-x}$ in $-1 < x < 1$. (8)

13. (a) A tightly stretched string with fixed end points $x = 0$ and $x = 50$ is initially at rest in its equilibrium position. If it is set to vibrate by giving each point a velocity $v = v_0 \sin\frac{\pi x}{50}\cos\frac{2\pi x}{50}$, find the displacement of any point of the string at any subsequent time. (16)

Or

(b) A rectangular plate with insulated surface is 10cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature of the short edge $y = 0$ is given by $u = \begin{cases} 20x, & 0 \le x \le 5 \\ 20(10 - x), & 5 < x \le 10 \end{cases}$ and the two long edges $x = 0$ and $x = 10$ as well as the other short edge are at 0°C. Find the steady state temperature at any point. (16)

14. (a) Show that the Fourier transform of $f(x)=\begin{cases} a^2-x^2, & |x|<a \\ 0, & |x|\geq a \end{cases}$ is $2\sqrt{\frac{2}{\pi}}\left(\frac{\sin as - as\cos as}{s^3}\right)$ hence find the value of $\int_0^\infty \frac{\sin t - t\cos t}{t^3}dt$. Also prove that $\int_0^\infty \left(\frac{\sin t - t\cos t}{t^3}\right)^2 dt = \frac{\pi}{15}$. (16)

Or

(b) (i) Evaluate $\int_0^\infty \frac{dx}{(x^2+1)(x^2+4)}$, using transform method. (8)

(ii) Solve the integral equation $\int_0^\infty f(x)\cos\lambda x\,dx = \begin{cases} 1-\lambda, & 0\leq\lambda\leq 1 \\ 0, & \lambda>1 \end{cases}$. (8)

15. (a) (i) State and prove convolution theorem of Z – transform. (8)

(ii) Find $Z^{-1}\left[\frac{z(z^2-z+2)}{(z+1)(z-1)^2}\right]$. (8)

Or

(b) (i) Find the Z – transforms of $\frac{1}{(n+1)}$ and $\frac{1}{(n+1)!}$. (8)

(ii) Solve $u_{n+2}-5u_{n+1}+6u_n=4^n$ given that $u_0=0, u_1=1$, using Z – transform. (8)

Part - A

1) p.d. w.r.to x & y, $x+a = p/2$, $y-b = q/2 \Rightarrow p^2+q^2 = 4z$.

2) A.E is $m^2+2m+1=0 \Rightarrow (m+1)^2=0 \Rightarrow m=-1,-1 \Rightarrow$ C.F $= \phi_1(y-x)+x\phi_2(y-x)$.

3) (i) $f(x)$ has a finite no. of discontinuities in any one period.

(ii) $f(x)$ has no or finite no. of maxima or minima.

4) $b_{10} = 0$.

5) $u(x,t) = \left(A_1 e^{\lambda x} + B_1 e^{-\lambda x}\right) e^{\lambda^2\alpha^2 t}$

$u(x,t) = \left(A_2 \cos\lambda x + B_2 \sin\lambda x\right) e^{-\lambda^2\alpha^2 t}$

$u(x,t) = \left(A_3 x + B_3\right)$

6) $B^2 - 4AC = -4x^2 < 0 \Rightarrow$ ellipse.

7) $f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t)\, e^{is(x-t)}\, dt\, ds$.

8) $F_c(s) = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos sx\, dx \Rightarrow \frac{d}{ds}F_c(s) = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x)(-\sin sx)x\, dx$.

$= -\sqrt{\frac{2}{\pi}}\int_0^\infty [xf(x)]\sin sx\, dx = -F_s[xf(x)]$.

9) $\delta(n) = \begin{cases} 1, & n=0 \\ 0, & n\neq 0 \end{cases} \Rightarrow Z[\delta(n)] = 1$.

10) If $Z[f(t)] = F(z)$, then $f(0) = \lim_{z\to\infty} F(z)$.

Part - B

11) a(i) The eqn. is $x^2+(y-a)^2+(z-b)^2 = c^2$.

p.d. w.r.to x & y, $z-b = -x/p$, $y-a = qx/p \Rightarrow (1+p^2+q^2)x^2 = c^2p^2$.

(ii) multipliers $(1,1,1) \Rightarrow x+y+z = c_1$

" $(1/x, 1/y, 1/z) \Rightarrow xyz = c_2$

$\Rightarrow \phi(x+y+z,\ xyz) = 0$.

b(i) $\sqrt{p}-2x = \sqrt{q} = k. \Rightarrow p^2 = (k+2x)^2$ & $q = k^2$.

W.k.t, $dz = p\,dx + q\,dy \Rightarrow$ Complete soln. $z = \frac{(k+2x)^3}{6} + k^2y + c$.

(ii) C.F $\phi_1(y+x)+\phi_2(y+2x)$. $P.I_1 = \frac{1}{D^2-3DD'+2D'^2} e^{3x+4y} = \frac{e^{3x+4y}}{5}$.

$P.I_2 = \frac{1}{D^2-3DD'+2D'^2}.\sin(4x-3y) = -\frac{\sin(4x-3y)}{70}$,

$z = C.F + P.I_1 + P.I_2$.

12) a(i) $\sin x$ is an odd func. $\Rightarrow a_0 = a_n = 0$.

$b_n = 0,\ n \neq 1,\ b_1 = 1. \Rightarrow f(x) = \sum_{n=1}^{\infty} b_n \sin nx = \sin x.$

(ii) $l = 3$.

$f(x) = \frac{a_0}{2} + a_1 \cos\frac{\pi x}{3} + a_2 \cos\frac{2\pi x}{3} + a_3 \cos \pi x + b_1 \sin\frac{\pi x}{3} + b_2 \sin\frac{2\pi x}{3} + b_3 \sin \pi x.$

Here, $a_0 = 14,\ a_1 = -2.83,\ a_2 = -1.5,\ a_3 = 2.67$

$b_1 = 4.04,\ b_2 = -0.87,\ b_3 = 0.$

b(i) $f(x) = \frac{a_0}{2} + \sum_1^{\infty} a_n \cos\frac{n\pi x}{l}$. where $a_0 = \frac{2}{l}\int_0^l f(x)\,dx,\ f(x) = x.$
$= l.$

$$a_n = \begin{cases} \frac{-4l}{n^2\pi^2}, & n \text{ is odd} \\ 0, & n \text{ is even} \end{cases}$$

(ii) $l = 1$. $f(x) = \sum_{-\infty}^{\infty} c_n e^{in\pi x}$ where $c_n = \left(\frac{1 - in\pi}{1 + n^2\pi^2}\right)(-1)^n \sinh 1.$

$\therefore f(x) = \sum_{-\infty}^{\infty} \left(\frac{1 - in\pi}{1 + n^2\pi^2}\right)(-1)^n \sinh 1 \cdot e^{in\pi x}.$

13) a) The bndy conditions are (i) $y(0,t) = 0$ (ii) $y(50,t) = 0$

(iii) $y(x,0) = 0$ (iv) $y_t(x,0) = v_0 \sin\frac{\pi x}{50} \cdot \cos\frac{2\pi x}{50}$.

The soln. is $y(x,t) = (A\cos\lambda x + B\sin\lambda x)(C\cos\lambda a t + D\sin\lambda a t)$

(i),(ii),(iii) $\Rightarrow A = 0,\ \lambda = \frac{n\pi}{50},\ C = 0.$

The most general soln is $y(x,t) = \sum_1^{\infty} b_n \sin\frac{n\pi x}{50} \cdot \sin\frac{n\pi a t}{50}$.

$\Rightarrow b_1 = \frac{-25 v_0}{\pi a},\ b_3 = \frac{25 v_0}{3\pi a},\ b_2 = b_4 = b_5 = \cdots = 0.$

b) Refer: C(181) 14 (b).

14) (a) $F(f(x)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx = 2\sqrt{\frac{2}{\pi}} \left(\frac{\sin as - as\cos as}{s^3}\right)$

$\Rightarrow f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) \cdot e^{-isx} ds = \frac{4}{\pi} \int_0^{\infty} \left(\frac{\sin as - as\cos as}{s^3}\right) \cos sx\, ds.$

Put $a = 1$ & $x = 0$, $\int_0^{\infty} \left(\frac{\sin t - t\cos t}{t^3}\right) dt = \pi/4$.

Using Parseval's, $\int_0^{\infty} \left(\frac{\sin t - t\cos t}{t^3}\right)^2 dt = \int_0^{\infty} (a^2 - x^2)^2 dx = \pi/15$.

(b)(i) Let $f(x) = e^{-x},\ g(x) = e^{-2x} \Rightarrow F_c(s) = \frac{1}{s^2+1},\ G_c(s) = \frac{2}{s^2+4}$.

$\therefore$ By P.I., $\int_0^{\infty} \frac{dx}{(x^2+1)(x^2+4)} = \pi/12$.

$\therefore \sqrt{\frac{2}{\pi}} \int_0^1 \sqrt{\frac{2}{\pi}} (1-x) \cos \lambda x \, dx = \frac{2}{\pi}\left[(1-x)\left(\frac{\sin \lambda x}{\lambda}\right) - (-1)\left(\frac{-\cos \lambda x}{\lambda^2}\right)\right]_0^1$

$= \frac{2}{\pi}\left[-\frac{\cos x}{x^2} + \frac{1}{x^2}\right] = \frac{2}{\pi}\left(\frac{1-\cos x}{x^2}\right).$

$Z[f(n)] = F(z)$, $Z[g(n)] = G(z)$ then

$[f(n) * g(n)] = F(z) \cdot G(z)$.

$) * g(n)] = Z\left[\sum_{k=-\infty}^{\infty} x(k) \cdot y(n-k)\right] = \sum_{n=-\infty}^{\infty}\left[\sum_{k=-\infty}^{\infty} x(k) y(n-k)\right] z^{-n}$

$\sum_{k=-\infty}^{\infty} x(k) \cdot z^{-k} \cdot \sum_{m=-\infty}^{\infty} y(m) z^{-m} = F(z) \cdot G(z)$

$\frac{z(z^2-z+2)}{(z+1)(z-1)^2}\Big] = Z^{-1}\left[\frac{z}{z+1} + \frac{z}{(z-1)^2}\right] = (-1)^n + n.$

$(1/n+1) = \sum_{n=0}^{\infty} \frac{1}{n+1} \cdot z^{-n} = z \log\left(z/z-1\right)$.

$\left(1/(n+1)!\right) = \sum_{n=0}^{\infty} \frac{1}{(n+1)!} z^{-n} = z\left(e^{1/z} - 1\right)$.

$u_{n+2} - 5u_{n+1} + 6u_n] = Z(4^n)$ with $u_0 = 0, u_1 = 1$.

$u_n) - u(0) - u(1)/z] - 5z[Z(u_n) - u(0)] + 6Z(u_n) = \frac{z}{z-4},$

$+6) Z(u_n) = \frac{z}{z-4} + z \Rightarrow Z(u_n) = \frac{z(z-3)}{(z-2)(z-3)(z-4)}$

$\Rightarrow u(n) = Z^{-1}\left[\frac{z}{(z-2)(z-4)}\right] = -2^{n-1} + 2^{2n-1}$