

NOTES AND FORMULAS OF MACROECONOMICS**LEC 23-45****Eco403****ENDOGENOUS GROWTH THEORY**

Production function for endogenous growth model can be written as:

$$Y = A K,$$

Where

A is the amount of output for each unit of capital (A is exogenous & constant).

Investment: sY

Depreciation: δK

Equation of motion for total capital: $\Delta K = s Y - \delta K$

Divide through by K and use $Y = A K$, get:

$$\frac{\Delta Y}{Y} = \frac{\Delta K}{K} = s A - \delta$$

If $s A > \delta$, then income will grow forever, and investment is the “engine of growth.” Here, the permanent growth rate depends on s . In Solow model, it does not.

A TWO-SECTOR MODEL

u = fraction of labor in research (u is exogenous)

- Manufacturing production function:
 $Y = F [K, (1-u) E L]$
- Research production function:
 $\Delta E = g (u) E$
- Capital accumulation:
 $\Delta K = s Y - \delta K$
- In the steady state, manufacturing output per worker and the standard of living grow at rate:

$$\Delta E/E = g (u).$$

Key variables are:

s : affects the level of income, but not its growth rate (same as in Solow model)

u : affects level and growth rate of income

TIME HORIZONS

- **Long run:** Prices are flexible, respond to changes in supply or demand

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• **Short run:** many prices are “sticky” at some predetermined level
The economy behaves much differently when prices are sticky.

AGGREGATE DEMAND

The aggregate demand curve shows the relationship between the price level and the quantity of output demanded. For an intro to the AD/AS model, we use a simple theory of aggregate demand based on the Quantity Theory of Money.

THE QUANTITY EQUATION AS AGGREGATE DEMAND

Quantity equation: $M V = P Y$

The money demand function it implies: $(M/P) d = k Y$,

Where

$V = 1/k$ = velocity.

For given values of M and V, these equations imply an inverse relationship between P and Y.

THE DOWNWARD-SLOPING AD CURVE

An increase in the price level causes a fall in real money balances (M/P), causing a decrease in the demand for goods & services.

SHIFTING THE AD CURVE

An increase in the money supply shifts the AD curve to the right.

AGGREGATE SUPPLY IN THE LONG RUN

In the long run, output is determined by factor supplies and technology:

$$\bar{Y} = F(\bar{K}, \bar{L})$$

Y is the full-employment or natural level of output,

LONG-RUN EFFECTS OF AN INCREASE IN MONEY

An increase in M shifts the AD curve to the right.

FROM THE SHORT RUN TO THE LONG RUN

Over time, prices gradually become “unstuck.” When they do, will they rise or fall?

In the short-run equilibrium, if	then over time, the price level will
$Y > \bar{Y}$	Rise
$Y < \bar{Y}$	Fall
$Y = \bar{Y}$	Remain constant

Predicted effects of the oil price shock:

Inflation ↑, Output ↓, Unemployment ↑ and then a gradual recovery.

THE KEYNESIAN CROSS

It is the simple closed economy model in which income is determined by expenditure. This model is presented by J.M. Keynes.

Notations:

I = planned investment

$E = C + I + G$ = planned expenditure

Y = real GDP = actual expenditure

Actual expenditure is the amount that households, firms and the government spend on goods and services; it equals the economy's gross domestic product (GDP).

Planned expenditure is the amount households, firms and the government would like to spend on goods and services.

ELEMENTS OF THE KEYNESIAN CROSS

Consumption function: $C = C(Y - T)$

Govt policy variables: $G = \bar{G}$, $T = \bar{T}$

For now, investment is exogenous: $I = \bar{I}$

Planned expenditure: $E = C(Y - \bar{T}) + \bar{I} + \bar{G}$

Equilibrium condition: Actual expenditure = Planned expenditure

$$Y = E$$

AN INCREASE IN GOVERNMENT PURCHASES

- SOLVING FOR ΔY**

Equilibrium condition $Y = C + I + G$

In changes form $\Delta Y = \Delta C + \Delta I + \Delta G$

Since I is exogenous $= \Delta C + \Delta G$

Because $\Delta C = MPC \times \Delta Y = MPC \times \Delta Y + \Delta G$

Collect terms with ΔY on the left side of the equals sign: $(1 - MPC) \times \Delta Y = \Delta G$

Finally, solve for ΔY :

$$\Delta Y = \left(\frac{1}{1 - MPC} \right) \times \Delta G$$

- THE GOVERNMENT PURCHASES MULTIPLIER**

The increase in income resulting from Rs.1 increase in G is known as government purchases multiplier. In this model, the G multiplier equals:

$$\frac{\Delta Y}{\Delta G} = \left(\frac{1}{1 - MPC} \right)$$

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• **WHY THE MULTIPLIER IS GREATER THAN 1?**

Initially, the increase in G causes an equal increase in Y: $\Delta Y = \Delta G$.

But $\uparrow Y \Rightarrow \uparrow C \Rightarrow$ further $\uparrow Y \Rightarrow$ further $\uparrow C \Rightarrow$ further $\uparrow Y$

So the final impact on income is much bigger than the initial ΔG .

AN INCREASE IN TAXES

Initially, the tax increase reduces consumption, and therefore E : so firms reduce output, and income falls toward a new equilibrium

• **SOLVING FOR ΔY**

Equilibrium condition in changes $\Delta Y = \Delta C + \Delta I + \Delta G$

I and G are exogenous = ΔC

$$= MPC \times (\Delta Y - \Delta T)$$

Solving for ΔY : $(1 - MPC) \times \Delta Y = -MPC \times \Delta T$

Final result:

$$\Delta Y = \left(\frac{-MPC}{1 - MPC} \right) \times \Delta T$$

• **THE TAX MULTIPLIER**

The change in income resulting from a \$1 increase in T is known as tax multiplier

$$\frac{\Delta Y}{\Delta T} = \left(\frac{-MPC}{1 - MPC} \right)$$

IS CURVE

A graph of all combinations of r and Y that result in goods market equilibrium is called IS curve i.e. Actual expenditure (output) = planned expenditure. The equation for the IS curve is:

$$Y = C(Y - \bar{T}) + I(r) + \bar{G}$$

DERIVING THE IS CURVE

$\downarrow r \Rightarrow \uparrow I \Rightarrow E \Rightarrow Y$

IS CURVE'S SLOPE

The IS curve is negatively sloped. A fall in the interest rate motivates firms to increase investment spending, which drives up total planned spending (E). To restore equilibrium in the goods market, output (actual expenditure, Y) must increase.

SHIFTING THE IS CURVE: ΔG

At any value of r , $\uparrow G \Rightarrow \uparrow E \Rightarrow \uparrow Y$...so the IS curve shifts to the right.

THE THEORY OF LIQUIDITY PREFERENCE

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John Maynard Keynes presented a simple theory in which the interest rate is determined by money supply and money demand.

The supply of real money balances is fixed:

$$(M/P)^s = \bar{M}/\bar{P}$$

Demand for real money balances:

$$(M/P)^d = L(r)$$

The interest rate adjusts to equate the supply and demand for money:

$$\bar{M}/\bar{P} = L(r)$$

HOW CENTRAL BANK RAISES THE INTEREST RATE

To increase r , Central Bank reduces M .

LM CURVE

Now let's put Y back into the money demand function:

$$(M/P)^d = L(r, Y)$$

The LM curve is a graph of all combinations of r and Y that equate the supply and demand for real money balances. The equation for the LM curve is:

$$\bar{M}/\bar{P} = L(r, Y)$$

LM CURVE'S SLOPE

The LM curve is positively sloped. An increase in income raises money demand. Since the supply of real balances is fixed, there is now excess demand in the money market at the initial interest rate. The interest rate must rise to restore equilibrium in the money market.

THE SHORT-RUN EQUILIBRIUM

The short-run equilibrium is the combination of r and Y that simultaneously satisfies the equilibrium conditions in the goods & money markets:

$$Y = C(Y - \bar{T}) + I(r) + \bar{G}$$

$$\bar{M}/\bar{P} = L(r, Y)$$

EQUILIBRIUM IN THE IS-LM MODEL

The IS curve represents equilibrium in the goods market.

$$Y = C(Y - \bar{T}) + I(r) + \bar{G}$$

The LM curve represents money market equilibrium

$$\bar{M}/\bar{P} = L(r, Y)$$

POLICY ANALYSIS WITH THE IS-LM MODEL

Policymakers can affect macroeconomic variables with fiscal policy: G and/or T and monetary policy: M . we can use the IS-LM model to analyze the effects of these policies.

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INTERACTION BETWEEN MONETARY & FISCAL POLICY

Model: monetary & fiscal policy variables (M , G and T) are exogenous.

Real world: Monetary policymakers may adjust M in response to changes in fiscal policy, or vice versa. Such interaction may alter the impact of the original policy change.

CENTRAL BANK'S RESPONSE TO $\Delta G > 0$

Suppose Government increases G . Possible central bank responses are:

1. RESPONSE 1: HOLD M CONSTANT

If Government raises G , the IS curve shifts right. If central bank holds M constant, then LM curve doesn't shift.

Results: $\Delta Y = Y_2 - Y_1$ $\Delta r = r_2 - r_1$

2. RESPONSE 2: HOLD r CONSTANT

If Government raises G , the IS curve shifts right. To keep r constant, central bank increases M , to shift the LM curve right.

Results: $\Delta Y = Y_2 - Y_1$ $\Delta r = 0$

3. RESPONSE 3: HOLD Y CONSTANT

If Government raises G , the IS curve shifts right. To keep Y constant, central Bank reduces M to shift LM curve left.

Results: $\Delta Y = 0$ $\Delta r = r_3 - r_1$

DERIVING THE AD CURVE

Intuition for slope of AD curve:

$P \Rightarrow \downarrow (M/P) \Rightarrow LM \text{ shifts left} \Rightarrow r \Rightarrow \downarrow I \Rightarrow \downarrow Y$

MONETARY POLICY AND THE AD CURVE

The central bank can increase aggregate demand:

$M \Rightarrow LM \text{ shifts right} \Rightarrow \downarrow r \Rightarrow I \Rightarrow Y \text{ at each value of } P$.

FISCAL POLICY AND THE AD CURVE

Expansionary fiscal policy (G and/or $\downarrow T$) increases aggregate demand:

$\downarrow T \Rightarrow C \Rightarrow IS \text{ shifts right} \Rightarrow Y \text{ at each value of } P$.

THE SR AND LR EFFECTS OF AN IS SHOCK

A negative IS shock shifts IS and AD left, causing Y to fall.

In the new short-run equilibrium, $Y < Y^*$.

Over time, P gradually falls, which causes $SRAS$ to move down, M/P to increase, which causes LM to move down.

• SHORT RUN IMPACTS

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Y	+	Because Y moved.
P	0	Because prices are sticky in the SR.
r	+	Because a $+\Delta Y$ leads to a rise in r as IS slides along the LM curve.
C	+	Because a $+\Delta Y$ increases the level of consumption ($C=C(Y-T)$).
I	-	Since r increased, the level of investment decreased.

• LONG RUN IMPACTS

Y	0	Because rising P shifts LM to left, returning Y to Y^* as required by long-run LRAS.
P	+	In order to eliminate the excess demand at P_0 .
r	+	Reflecting the leftward shift in LM due to $+\Delta P$.
C	0	Since both Y and T are back to their initial levels ($C=C(Y-T)$).
I	--	Since r has risen even more due to the $+\Delta P$.

ANALYZE SR & LR EFFECTS OF ΔM

We have IS-LM and AD-AS diagrams as shown here. Suppose central bank increases M.

• SHORT RUN IMPACTS

Y	+	Because Y moved.
P	0	Because prices are sticky in the SR.
r	-	Because a $+\Delta Y$ leads to a decrease in r as LM slides along the IS curve.
C	+	Because a $+\Delta Y$ increases the level of consumption ($C=C(Y-T)$).
I	+	Since r decreased, the level of investment increased

• LONG RUN IMPACTS

Y	0	Because rising P shifts LM to left, returning Y to Y^* as required by long-run LRAS.
P	+	In order to eliminate the excess demand at P_0 .
r	0	Reflecting the leftward shift in LM due to $+\Delta P$ restoring r to its original level.
C	0	Since both Y and T are back to their initial levels ($C=C(Y-T)$).
I	0	Since Y or r has not changed.

Notice that the only LR impact of an increase in the money supply was an increase in the price level.

THE MUNDELL-FLEMING MODEL

The Mundell-Fleming model portrays the relationship between the nominal exchange rate and the economy output. It is an extension of IS-LM model. Key assumption of this model is the small open economy with perfect capital mobility.

$$r = r^* \text{ (given)}$$

IS* CURVE: GOODS MARKET EQUILIBRIUM

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Goods market equilibrium-the IS* curve:

$$Y = C(Y - T) + I(r^*) + G + NX(e)$$

Where:

e = nominal exchange rate = foreign currency per unit of domestic currency (e.g. 110 yen per dollar). The IS* curve is drawn for a given value of r^* .

Intuition for the slope:

$$\downarrow e \Rightarrow \uparrow NX \Rightarrow \uparrow Y$$

LM* CURVE: MONEY MARKET EQUILIBRIUM

The LM* curve equation is:

$$MP = L(r^*, Y)$$

LM* curve is drawn for a given value of r^* . It is vertical because given r^* , there is only one value of Y that equates money demand with supply, regardless of e .

FISCAL POLICY UNDER FLOATING EXCHANGE RATES

$$Y = C(Y - T) + I(r^*) + G + NX(e)$$

$$MP = L(r^*, Y)$$

At any given value of e , a fiscal expansion increases Y , shifting IS* to the right.

Results: $\Delta e > 0, \Delta Y = 0$

MONETARY POLICY UNDER FLOATING EXCHANGE RATES

An increase in M shifts LM* right because Y must rise to restore equilibrium in the money market.

Results: $\Delta e < 0, \Delta Y > 0$

LESSONS ABOUT MONETARY POLICY

Monetary policy affects output by affecting one (or more) of the components of aggregate demand:

Closed economy: $\uparrow M \Rightarrow \downarrow r \Rightarrow \uparrow I \Rightarrow \uparrow Y$

Small open economy: $\uparrow M \Rightarrow \downarrow e \Rightarrow \uparrow NX \Rightarrow \uparrow Y$

Expansionary monetary policy does not raise world aggregate demand, it shifts demand from foreign to domestic products. Thus, the increases in income and employment at home come at the expense of losses abroad.

TRADE POLICY UNDER FLOATING EXCHANGE RATES

At any given value of e , a tariff or quota reduces imports, increases NX , and shifts IS* to the right.

- a. The Equilibrium exchange rate is Greater than the fixed exchange rate
- b. The Equilibrium exchange rate is less than the fixed exchange rate

FISCAL POLICY UNDER FIXED EXCHANGE RATES

Under fixed exchange rates, a fiscal expansion would raise e . To keep e from rising, the central bank must sell domestic currency, which increases M and shifts LM^* right.

Results: $\Delta e = 0, \Delta Y > 0$.

Under floating rates, fiscal policy ineffective at changing output. Under fixed rates, fiscal policy is very effective at changing output. LM shifts out!

MONETARY POLICY UNDER FIXED EXCHANGE RATES

An increase in M would shift LM^* right and reduce e . To prevent the fall in e , the central bank must buy domestic currency, which reduces M and shifts LM^* back left.

Results: $\Delta e = 0, \Delta Y = 0$.

Under floating rates, monetary policy is very effective at changing output. Under fixed rates, monetary policy cannot be used to affect output.

TRADE POLICY UNDER FIXED EXCHANGE RATES

A restriction on imports puts upward pressure on e . To keep e from rising, the central bank must sell domestic currency, which increases M and shifts LM^* right.

Results: $\Delta e = 0, \Delta Y > 0$.

Under floating rates, import restrictions do not affect Y or NX . Under fixed rates, import restrictions increase Y and NX . But, these gains come at the e

M-F: SUMMARY OF POLICY EFFECTS

	Type of Exchange Rate Regime					
	Floating			Fixed		
	Impact on					
Policy	Y	e	NX	Y	e	NX
Fiscal Expansion	0	+	-	+	0	0
Monetary Expansion	+	-	+	0	0	0
Import Restriction	0	+	0	+	0	+

DIFFERENTIALS IN THE M-F MODEL

$$r = r^* + \theta$$

Where θ is a risk premium. Substitute the expression for r into the IS^* and LM^* equations:

$$Y = C(Y - T) + I(r^* + \theta) + G + NX(e)$$

$$MP = L(r^* + \theta, Y)$$

THE EFFECTS OF AN INCREASE IN θ

IS* shifts left, because $\uparrow \theta \Rightarrow \uparrow r \Rightarrow \downarrow I$

LM* shifts right, because $\uparrow \theta \Rightarrow \uparrow r \Rightarrow \downarrow (M/P) d$, So Y must rise to restore money market equilibrium.

THE EFFECTS OF AN INCREASE IN θ

The fall in e is intuitive: An increase in country risk or an expected depreciation makes holding the country's currency less attractive.

Note: an expected depreciation is a self-fulfilling prophecy. The increase in Y occurs because the boost in NX (from the depreciation) is even greater than the fall in I (from the rise in r).

MUNDALL-FLEMING AND THE AD CURVE

Previously, we examined the M-F model with a fixed price level. To derive the AD curve, we now consider the impact of a change in P in the M-F model. We now write the M-F equations as:

$$(IS^*) Y = C(Y - T) + I(r^*) + G + NX(\epsilon)$$

$$(LM^*) M/P = L(r^*, Y)$$

DERIVING THE AD CURVE

AD curve has negative slope because:

As $P \Rightarrow \downarrow (M/P) \Rightarrow LM$ shifts left $\Rightarrow \epsilon \Rightarrow \downarrow NX \Rightarrow \downarrow Y$

FROM SHORT RUN TO THE LONG RUN

If $Y_1 < Y$ then there is downward pressure on prices. Over time, P will move down, causing

$(M/P) \Rightarrow \epsilon \downarrow \Rightarrow NX \Rightarrow Y$

LARGE: BETWEEN SMALL AND CLOSED

Many countries - including the U.S. - are neither closed nor small open economies. A large open economy is in between the polar cases of closed & small open. Consider a monetary expansion:

Like in a closed economy,

$\Delta M > 0 \Rightarrow \downarrow r \Rightarrow \uparrow I$ (though not as much)

Like in a small open economy,

$\Delta M > 0 \Rightarrow \downarrow \epsilon \Rightarrow \uparrow NX$ (though not as much)

THREE MODELS OF AGGREGATE SUPPLY**1. The sticky-wage model:**

The nominal wage, W, they set is the product of a target real wage, ω , and the expected price level:

$$W = \omega \times P^e$$

$$\Rightarrow \frac{W}{P} = \omega \times \frac{P^e}{P}$$

If	Then
$P = P^e$	Unemployment and output are at their natural rates
$P > P^e$	Real wage is less than its target, so firms hire more workers and output rises above its natural rate
$P < P^e$	Real wage exceeds its target, so firms hire fewer workers and output falls below its natural rate

- In booms, when P typically rises, the real wage should fall.
- In recessions, when P typically falls, the real wage should rise.

2. The imperfect-information model

Assumptions:

- All wages and prices perfectly flexible.
- All markets are clear.
- Each supplier produces one good, consumes many goods.
- Each supplier knows the nominal price of the good she produces, but does not know the overall price level.

3. The sticky-price model

Reasons for sticky prices are as follows:

- Long-term contracts between firms and customers.
- Menu costs.
- Firms do not wish to annoy customers with frequent price changes.

Assumptions:

Firms set their own prices (e.g. as in monopolistic competition).

An individual firm's desired price is

$$p = P + \alpha (Y - \bar{Y})$$

Where $\alpha > 0$.

Suppose two types of firms:

- Firms with flexible prices, set prices as above
- Firms with sticky prices must set their price before they know how P and Y will turn out:

$$p = P^e + \alpha (Y^e - \bar{Y}^e)$$

Assume firms with sticky prices expect that output will equal its natural rate. Then,

$$p = P^e$$

To derive the aggregate supply curve, we first find an expression for the overall price level.

Let s denote the fraction of firms with sticky prices. Then, we can write the overall price level as

$$P = s P^e + (1 - s) \left[P + \alpha Y - \bar{Y} \right]$$

Price set by sticky price firm Price set by flexible price firm

Subtract $(1-s)P$ from both sides:

$$sP = sP^e + (1 - s) \left[P + \alpha Y - \bar{Y} \right]$$

Divide both sides by s :

$$P = P^e + \left[\frac{(1 - s)\alpha}{s} \right] Y - \bar{Y}$$

High $Y \Rightarrow$ High P . When income is high; the demand for goods is high. Firms with flexible prices set high prices. The greater the fraction of flexible price firms, the smaller is s and the bigger is the effect of ΔY on P .

Finally, derive AS equation by solving for Y :

$$Y = \bar{Y} + \alpha(P - P^e)$$

$$\text{Where: } \alpha = \frac{s}{1 - s}$$

All three models imply: $Y = \bar{Y} + \alpha(P - P^e)$

Where:

Y Aggregate output

\bar{Y} Natural rate of output

α a positive parameter

P the actual price level

P^e The expected price level

INFLATION, UNEMPLOYMENT, AND THE PHILLIPS CURVE

The Phillips curve states that π depends on Expected inflation, π^e

Cyclical unemployment: the deviation of the actual rate of unemployment from the natural rate

Supply shocks, v

$$\pi = \pi^e - \beta(u - u^n) + v$$

Where $\beta > 0$ is an exogenous constant.

DERIVING THE PHILLIPS CURVE FROM SRAS

1- We can drive the Philips curve from our equation for aggregate supply.

$$Y = \bar{Y} + \alpha(P - P^e)$$

2- According to aggregate supply equation:

$$P = P^e + \frac{1}{\alpha} (Y - \bar{Y})$$

3- Add to the right-hand side of the equation a supply shock v that alter the price level and shift the short run aggregate supply curve:

$$P = P^e + \frac{1}{\alpha} (Y - \bar{Y}) + v$$

4- Subtract last year's price level P_{-1} from both sides of equation to obtain

$$P - P_{-1} = P^e - P_{-1} + \frac{1}{\alpha} (Y - \bar{Y}) + v$$

5- The term on the left hand side is the difference between current price level and last years price level, which is inflation. The term on the right hand side is the difference between the expected price level and last years price level, which is expected inflation. Therefore,

$$\pi = \pi^e + \frac{1}{\alpha} (Y - \bar{Y}) + v$$

6- Okun's law which gives a relationship between two variables. We can write this as

$$\frac{1}{\alpha} (Y - \bar{Y}) = -\beta(u - u^n)$$

7- Using this Okun's law relationship, we can substitute left-hand side value in equation number and we obtain

$$\pi = \pi^e - \beta(u - u^n) + v$$

THE PHILLIPS CURVE AND SRAS

SRAS : $Y = \bar{Y} + \alpha(P - P^e)$

PHILLIPS CURVE : $\pi = \pi^e - \beta(u - u^n) + v$

SRAS curve:

Output is related to unexpected movements in the price level.

Phillips curve:

Unemployment is related to unexpected movements in the inflation rate.

ADAPTIVE EXPECTATIONS

Adaptive expectations: an approach that assumes people form their expectations of future inflation based on recently observed inflation. A simple example:

Expected inflation = last year's actual inflation:

$$\pi^e = \pi_{-1}$$

Then, the Philips curve becomes:

$$\pi = \pi_{-1} - \beta(u - u^n) + v$$

GRAPHING THE PHILLIPS CURVE

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In the short run, policymakers face a trade-off between π and u .

SHIFTING THE PHILLIPS CURVE

People adjust their expectations over time, so the tradeoff only holds in the short run. e.g., an increase in π^e shifts the short-run P.C. upward.

THE SACRIFICE RATIO

To reduce inflation, policymakers can contract aggregate demand, causing unemployment to rise above the natural rate. The sacrifice ratio measures the percentage of a year's real GDP that must be foregone to reduce inflation by 1 percentage point. Its estimates vary, but a typical one is 5.

RATIONAL EXPECTATIONS

Ways of modeling the formation of expectations:

- **Adaptive expectations:** People base their expectations of future inflation on recently observed inflation.
- **Rational expectations:** People base their expectations on all available information, including information about current & prospective future policies.

HYSTERESIS: the long-lasting influence of history on variables such as the natural rate of unemployment. Negative shocks may increase u_n , so economy may not fully recover.

GOVERNMENT DEBT AND THE ANNUAL BUDGET DEFICIT

When a government spends more than it collects in taxes, it borrows from the private sector to finance the budget deficit. The government debt is an accumulation of all past annual deficits.

• PROBLEMS IN MEASUREMENT

Govt. Budget Deficit = Govt. Spending – Govt. Revenue
= Amount of new debt

• INFLATION

The commonly measured budget deficit does not correct for inflation.

Suppose the real government debt is not changing. In other words, in real terms, the budget is balanced. In this case, the nominal debt must be rising at the rate of inflation. i.e.

$$\Delta D / D = \pi$$

Where, π is the inflation rate and D is the stock of government debt. This implies

$$\Delta D = \pi D$$

So by looking at the change in nominal debt ΔD , a budget deficit of πD can be reported.

Another perspective: Govt. budget deficit = govt. Expenditure – Govt. Revenues

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For correct measurement of budget deficit, the government expenditure should include only the real interest paid on the debt (rD), not the nominal interest paid (iD).

$$\text{Since, } i - r = \pi$$

Budget deficit is overstated by ΠD

Example:

In 1979, Budget deficit = \$28 billions, $\pi = 8.6\%$, Government debt = \$495 billion

Budget Deficit overstated, $\pi D = 0.086 \times 495 = \43 billion

So, \$28 - \$43 = \$15 billion surplus

CAPITAL ASSETS

An accurate assessment of government's budget deficit requires accounting for the govt.'s assets as well as liabilities.

$$\text{Govt. budget deficit} = \text{change in debt} - \text{change in assets.}$$

THE BUSINESS CYCLE

Changes occur automatically in response to a fluctuating economy.

Example: Recession

Low Incomes \Rightarrow Low Personal Taxes

Low Profits \Rightarrow Low Corporate Taxes

High Number of needy person's \Rightarrow High G, Budget Deficit Increases

SOLOW GROWTH MODEL

Change in capital stock = investment - depreciation

$$\Delta k = i - \delta k$$

Since $i = sf(k)$, this becomes: $\Delta k = sf(k) - \delta k$

STARTING WITH TOO LITTLE CAPITAL

If $K^* < K^*$ gold, then increasing c^* requires an increase in s .

A TAX CUT

We Have $C = C(Y - T)$, at any value of r , $\downarrow T \Rightarrow \uparrow C \Rightarrow \uparrow E \Rightarrow \uparrow Y$

So the IS curve shifts to the right. The horizontal distance of the IS shift equals

$$\Delta Y = \text{MPC} / (1 - \text{MPC}) \Delta T.$$

MUNDELL-FLEMING MODEL

$$Y = C(Y - T) + I(r^*) + G + N X(e)$$

$$MP = L(r^*, Y)$$

At any given value of e , a fiscal expansion increases Y , shifting IS^* to the right.

Results:

$$\Delta e > 0, \Delta Y = 0.$$

THE CONSUMPTION FUNCTION

$$C = \bar{C} + cY$$

Consumption
Depends
Autonomous
Marginal
Income

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Householdson Consumption
Consume (MPC)

Propensity to

AVERAGE PROPENSITY TO CONSUME

$APC = C/Y = C/Y + c$, As Y rises, C/Y falls, and so the average propensity to consume C/Y falls.

MARGINAL PROPENSITY TO CONSUME

The MPC measures the sensitivity of the change in one variable (C) with respect to a change in the other variable (Y).

CONSUMPTION PUZZLE

Studies of household data and short time-series found a relationship between consumption and income similar to the one Keynes conjectured-- this is called the **short-run consumption function**.

But, studies using long time-series found that the APC did not vary systematically with income--this relationship is called the **long-run consumption function**.

CONSUMER'S BUDGET CONSTRAINT

Consider the decision facing a consumer who lives for two periods (representing youth & age).

He earns Income Y_1 , Y_2 and consumes C_1 , C_2 in both periods respectively (adjusted for inflation). The savings in the first period will be

$$S = Y_1 - C_1$$

In the second period

$$C_2 = (1 + r) S + Y_2$$

Where r is the real interest rate. Remember S can represent either saving or borrowing and the equations hold in both cases.

- If $C_1 < Y_1$ consumer is saving $S > 0$
- If $C_1 > Y_1$ consumer is borrowing $S < 0$

Assume: r (borrowing) = r (saving)

Combining the two equations:

$$C_2 = (1 + r) (Y_1 - C_1) + Y_2$$

Rearranging

$$(1 + r)C_1 + C_2 = (1 + r) Y_1 + Y_2$$

Dividing both sides by $1 + r$

$$C_1 + \frac{C_2}{1 + r} = Y_1 + \frac{Y_2}{1 + r}$$

THE LIFE-CYCLE CONSUMPTION FUNCTION

The Lifetime resources of consumer for T years are wealth W and lifetime earnings of

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$R \times Y$ (assuming interest rate to be zero). To have smoothest consumption over lifetime, she divides such that:

$$C = (W + RY) / T \text{ or } C = (1 / T) W + (R / T) Y$$

If she expects $T = 50$ and $R = 30$, then the consumption function will be

$$C = 1 / 50 W + 30 / 50 Y \text{ or}$$

$$C = 0.02W + 0.6Y$$

Generalizing for Aggregate Consumption function of the economy:

$$C = \alpha W + \beta Y$$

Where,

α = MPC out of Wealth

β = MPC out of Income

SOLVING THE CONSUMPTION PUZZLE

According to Life-cycle consumption function,

$$APC = C/Y = \alpha (W/Y) + \beta$$

Because, in short periods, wealth does not vary proportionately with incomes, High incomes correspond to Low APC. But over longer periods, wealth and incomes grow together, resulting in constant W/Y ratio and hence a constant APC.

MILTON FRIEDMAN AND THE PERMANENT-INCOME HYPOTHESIS

Friedman suggested that we view current income Y as the sum of two components, permanent income Y_P and transitory income Y_T .

$$Y = Y_P + Y_T$$

Friedman approximation of consumption function is:

$$C = \alpha Y_P$$

While Average propensity to consume is:

$$APC = C/Y = \alpha Y_P / Y$$

- When $Y > Y_P$, APC Falls
- When $Y < Y_P$, APC rises

Investment is the most volatile component of GDP. When expenditure on goods and services fall during a recession, much of the decline is usually due to a drop in investment spending.

The models of GDP, such as IS-LM model, were based on a simple investment function relating investment to real interest rate:

$$I = I(r).$$

That function states that an increase in the real interest rate reduces Investment.

BUSINESS FIXED INVESTMENT

The largest piece of investment spending (about $\frac{3}{4}$ of total) is business fixed investment.

- Business: these investment goods are bought by firms for use in future production.

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- Fixed: This spending is for capital that will stay put for a while (as opposed for inventory investment)

THE RENTAL PRICE OF CAPITAL

A typical production firm decides how much capital to rent by comparing the cost and benefit of each unit of capital.

To maximize profit, the firm rents capital until the MPK falls to:

$$\text{MPK} = R/P$$

Hence MPK determines the downward sloping demand curve for capital for a firm. While at point in time, the amount of capital in an economy is fixed, so supply curve is fixed.

The real rental price of capital adjusts to equilibrate the demand for capital and the fixed supply.

The Cobb-Douglas production function serves as a good approximation of how the actual economy turns capital and labor into goods and services. The Cobb-Douglas production function is:

$$Y = AK^\alpha L^{(1-\alpha)}$$

Where,

$Y \Rightarrow$ is output

$K \Rightarrow$ capital

$L \Rightarrow$ labor

$A \Rightarrow$ a parameter measuring the level of technology

$\alpha \Rightarrow$ a parameter between 0 and 1 that measures capital's share of output.

The marginal product of capital (MPK) for the Cobb-Douglas production function is:

$$\text{MPK} = \alpha A (L/K)^{1-\alpha}$$

Because the real rental price (R/P) equals MPK in equilibrium, we can write:

$$R/P = \alpha A (L/K)^{1-\alpha}$$

This expression identifies the variables that determine the real rental price.

THE COST OF CAPITAL

For each period of time that a firm rents out a unit of capital, the rental firm bears three costs:

1. Interest on their loans, which equals the purchase price of a unit of capital P_K times the interest rate, i , so iP_K
2. The cost of the loss or gain on the price of capital denoted as $-\Delta P_K$
3. Depreciation δ defined as the fraction of value lost per period because of the wear and tear, so δP_K

Therefore, Total cost of capital

$$= iP_K - \Delta P_K + \delta P_K$$

Or

$$= P_K (i - \Delta P_K / P_K + \delta)$$

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Example: A Car rental company buys cars for Rs.1, 000,000 each and rents them out to other businesses. If it faces an interest rate i of 10% p.a. so the interest cost, $i P_K = \text{Rs.}100,000$ p.a.

Car prices are rising @ 6% per year, so excluding maintenance costs the firm gets a capital gain, $\Delta P_K = \text{Rs.}60,000$ p.a

Cars depreciate @ 20% p.a. so loss due to wear and tear, $\delta P_K = \text{Rs.}200,000$

$$\begin{aligned}\text{So, total cost of capital} &= i P_K - \Delta P_K + \delta P_K \\ &= 100,000 - 60,000 + 200,000 \\ &= \text{Rs.}240,000\end{aligned}$$

Assuming price of capital goods rises with the prices of other goods, so

$$\Delta P_K / P_K = \text{overall inflation rate, } \pi$$

Since,

$$r = i - \pi,$$

$$\text{Cost of Capital} = P_K (r + \delta)$$

To express the cost of capital relative to other goods in the economy. The real cost of capital-- the cost of buying and renting out a unit of capital measured in terms of the economy's output is:

$$\text{Real Cost of Capital} = (P_K / P) (r + \delta)$$

Where

$r \Rightarrow$ the real interest rate

$P_K / P \Rightarrow$ the relative price of capital.

THE DETERMINANTS OF INVESTMENT

The real profit per unit of capital is

$$\begin{aligned}\text{Profit rate} &= \text{Revenue} - \text{Cost} \\ &= R/P - (P_K / P) (r + \delta).\end{aligned}$$

Because real rental price equals the marginal product of capital, we can write the profit rate as

$$\text{Profit rate} = MPK - (P_K / P) (r + \delta)$$

The change in the capital stock, called net investment depends on the difference between the MPK and the cost of capital.

- If the MPK exceeds the cost of capital, firms will add to their capital stock.
- If the MPK falls short of the cost of capital, they let their capital stock shrink.

Thus:

$$\Delta K = \text{In} [MPK - (P_K / P) (r + \delta)]$$

Where $\text{In} ()$ is the function showing how much net investment responds to the incentive to invest.

THE INVESTMENT FUNCTION

The investment function is:

$$I = \text{In} [MPK - (P_K / P) (r + \delta)] + \delta K$$

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When the capital stock reaches a steady state level, we can write:

$$MPK = (PK / P) (r + \delta)$$

Corporate income tax

The law says:

$$\text{Profit rate} = R/P - (PK / P) (r + \delta)$$

THE STOCK MARKET AND TOBIN'S q

Tobin's q:

$$q = \frac{\text{Market Value of Installed Capital}}{\text{Replacement Cost of Installed Capital}}$$

THE ACCELERATOR MODEL OF INVENTORIES

If N is the economy's stock of inventories and Y is output, then

$$N = \beta Y$$

Where β is a parameter reflecting how much inventory firms wish to hold as a proportion of output. Inventory investment I is the change in the stock of inventories βN .

Therefore,

$$I = \Delta N = \beta \Delta Y$$

MONEY SUPPLY

Money supply includes both currency in the hand of public and deposits at banks that households use on demand for transactions.

$$M = C + D$$

Where

M ---> Money Supply

C ---> Currency

D ---> Demand Deposits

MONEY DEMAND

CLASSICAL THEORY OF MONEY DEMAND

The Quantity Theory of Money assumes that the demand for real money balances is directly proportional to income,

$$M / P_d = kY$$

Where k is a constant measuring how much people want to hold for every dollar of income.

KEYNESIAN THEORY OF MONEY DEMAND

It presents a more realistic money demand function where the demand for real money balances depends on i and Y :

$$M/P_d = L(i, Y)$$

PORTFOLIO THEORIES OF MONEY DEMAND

The money demand function as

$$M/P_d = L(r_s, r_b, \pi^e, W)$$

Where

r_s = expected real return on stock

r_b = expected real return on bonds

π^e = expected inflation rate

W = real wealth

Money Demand Function $L(i, Y)$: A useful simplification:

- Uses real income Y as proxy for real wealth W
- Nominal interest rate $i = r_b + \pi^e$

A MODEL OF MONEY SUPPLY

Three exogenous variables:

- The **monetary base B** is the total number of dollars held by the public as currency C and by the banks as reserves R .
- The **reserve-deposit ratio rr** is the fraction of deposits D that banks hold in reserve R .
- The **currency-deposit ratio cr** is the amount of currency C people hold as a fraction of their holdings of demand deposits D .

Definitions of money supply and monetary base:

$$M = C + D$$

$$B = C + R$$

Solving for M as a function of 3 exogenous variables:

$$M/B = C/D + 1$$

$$C/D + R/D$$

Making substitutions for the fractions above, we obtain:

$$M = \frac{cr + 1}{cr + rr} \times B$$

So

$$M = m \times B$$

Because the monetary base has a multiplied effect on the money supply, the monetary base is sometimes called *high-powered money*.

An Example

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Suppose, monetary base B is \$500 billion, the reserve deposit ratio rr is 0.1 and currency deposit ratio cr is 0.6

The money multiplier is:

$$m = \frac{0.6 + 1}{0.6 + 0.1} = 2.3$$

And the money supply is:

$$M = 2.3 \times \$ 500 \text{ billion} = \$1,150 \text{ billion}$$